AQA

A-LEVEL
Mathematics

MM2B Mechanics 2B
Mark scheme

6360

June 2018

Version/Stage: 1.0 Final

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Annotations key

Annotation	**Use**
?	Unclear
^	Omission
BOD	Benefit of doubt
Cross	Incorrect
FT	Follow through
H wavy	Highlight relevant working
ISW	Ignore subsequent working
On page comment	Clarifies issue
SEEN	Only used on blank pages
Tick	Correct

Q	Solution	Mark	Total	Comment
1 (a)(i)	Total weight is 20kg.	**B1**		B1: Total weight CAO
	Taking moments about y axis			
	$7\times60 + 6 \times 20+3 \times60 + 4\times 80 = 20x$	**M1**		M1: At least three correct multiplications on LHS.
	$x = \frac{1040}{20}$			
	$= 52$			
	Distance from AD is 52 cm	**A1**	**3**	A1: Correct distance CAO.
(ii)				
	Taking moments about x axis			
	$7\times 40 + 6\times60 + 3 \times30 + 4 \times 20 = 20y$	**M1**		M1: At least three correct multiplications on LHS.
	$y = \frac{810}{20}$			
	$= 40.5$			
	Distance from AB is 40.5 cm	**A1**	**2**	A1: Correct distance CAO.
(b)	If lamina hangs in equilibrium ,			
	C of G must be vertically below X			M1: Uses 60 or other appropriate distance in their calculations. No need to see this explicitly stated.
	Hence distance of C of G from y axis is 60 cm.	**M1**		
	Moments about AD;			
	$1040 + m \times 120 = (20 + m)\, 60$	**M1A1**		M1: At least two terms correct in a 3/4 term moment equation correct.
	$60\ m = 160$			A1: Correct moment equation.
	Mass is $\frac{8}{3}$ kg	**A1**	**4**	A1: Correct mass. CAO Accept 2.67.
	Total		**9**	

Q	Solution	Mark	Total	Comment
2 (a)	Initial KE is $\frac{1}{2} \times 21 \times 2^2$	**M1**		M1: Correct expression for KE.
	= 42 J	**A1**	**2**	A1: Correct value for KE
(b)(i)	Energy gained by moving to point B is mgh	**M1**		M1: For height change 8.
	$= 21 \times 8 \times g$			
	= 1646.4	**A1**		A1: Correct change in PE.
	Total KE at B is 1646.4 + 42			
	= 1688.4			
	= 1690 J	**A1F**	**3**	A1F: Correct sum of their energies.
(ii)	KE at point B is 1688.4			
	$\frac{1}{2} \times m \times v^2 = 1688.4$	**M1**		M1: Seeing $\frac{1}{2}mv^2$ equated to their answer to (b)(i).
	$v^2 = \frac{1688.4}{10.5}$			
	= 12.68...			
	Speed is 12.7 ms^{-1}	**A1F**	**2**	A1: Correct speed for their answer to (b)(i).
(c)	Work done = F × s = change in KE			
	$21g\mu s = 1688.4$	**M1**		M1: For *Fs* = Change in KE using their answer to (b)(i).
	$\mu = \frac{1688.4}{21 \times g \times 18}$	**A1F**		A1: Correct equation (using bi).
	= 0.45578..			
	= 0.456	**A1**	**3**	A1: Correct coefficient of friction. CAO
	OR			
	$a = -4.4666$	(M1)		M1: Correct acceleration (Accept -4.48 from 12.7)
	$-21g\mu = 21 \times (-4.4666)$	(A1)		A1: Correct equation.
	$\mu = \frac{4.4666}{g} = 0.456$	(A1)		A1: Correct coefficient of friction. CAO
	Total		**10**	

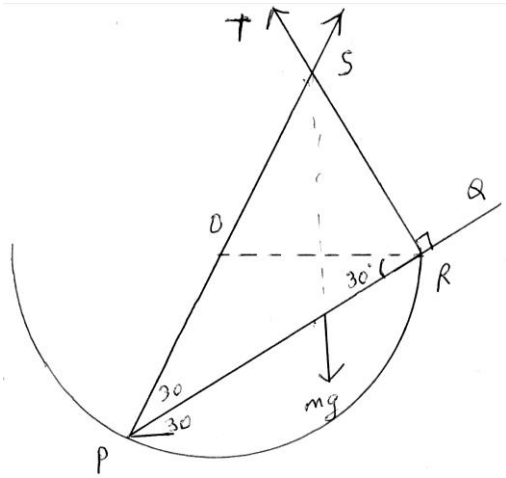
Q 3	Solution	Mark	Total	Comment
(a)	$\mathbf{a} = (12 - 3t^2)\mathbf{i} + 12\,e^{-2t}\mathbf{j}$	**M1A1**	**2**	M1: Either term correct. A1: All correct
(b)(i)	$\mathbf{F} = m\mathbf{a}$	**M1**		M1: Use of $F = ma$
	$= (24 - 6t^2)\mathbf{i} + 24\,e^{-2t}\mathbf{j}$	**A1**	**2**	A1: Correct expression for force.
(ii)	When t = 0, $\mathbf{F} = 24\mathbf{i} + 24\mathbf{j}$			
	Magnitude is $\sqrt{24^2 + 24^2}$	**M1**		M1: Finding magnitude and substituting $t = 0$.
	$= 24\sqrt{2}$ or 33.9	**A1**	**2**	A1: Correct magnitude. Do not only 34.
(c)	When **F** acts north, **i** component is zero			
	$24 - 6t^2 = 0$	**M1**		M1: Setting **i** component equal to zero.
	$t = 2$	**A1**	**2**	A1: Correct time.
(d)	$\mathbf{r} = \int \boldsymbol{v}\, dt$			
	$=(6t^2 - \frac{1}{4}\,t^4)\boldsymbol{i} + 3\,e^{-2t}\mathbf{j} + \boldsymbol{c}$	**M1A1**		M1: Either component correct. A1: Both components correct. Condone missing **c**.
	When t =0, $\mathbf{r} = 4\mathbf{i} - 2\,\mathbf{j}$, $\therefore \boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j}$	**m1A1**		m1: Must use t=0. Either component of **c** correct. A1: Correct **c**.
	$\mathbf{r} = (6t^2 - \frac{1}{4}\,t^4 + 4)\boldsymbol{i} + (3\,e^{-2t} - 5)\mathbf{j}$	**A1**	**5**	A1: Correct position vector.
	Total		**13**	

Q	Solution	Mark	Total	Comment
4 (a)	Using $F = \frac{mv^2}{r}$ $F = \frac{900\times12^2}{80}$ $= 1620$ N	**M1** **A1**	 **2**	M1: Using $F = \frac{mv^2}{r}$ A1: Correct force.
(b)	Using $F = \mu R$ $1620 = \mu \times 900g$ $\mu = 0.18367..$ $\mu = 0.184$	**M1** **A1**	 **2**	M1: Use of $F = \mu R$ Condone use of inequality. A1: Correct coefficient of friction.
	Total		**4**	

Q	Solution	Mark	Total	Comment
5(a)	Power = $F \times v$ $32000 = (k \times 40) \times 40$ $= 1600k$ $k = 20$	**M1** **A1**	 **2**	M1: Use of $P = Fv$ A1: Using an equation that leads to $k = 20$. AG
(b)(i)	Accelerating forces $= 600g \times \frac{1}{10} - 20v$ $\rightarrow \ 600\frac{dv}{dt} = 60g - 20v$ $\frac{dv}{dt} = \frac{g}{10} - \frac{v}{30}$ $\frac{dv}{dt} = \frac{3g-v}{30}$	 **M1** **A1** **A1**	 **3**	 M1: Resolving to find component of weight in equation. Must see $\sin\theta$ or $\times \frac{1}{10}$. A1: Use of $F = ma$ A1: Correct answer from correct working. AG
(ii)	$\int \frac{dv}{3g-v} = \frac{1}{30}\int dt$ $-\ln(3g - v) = \frac{1}{30}t + c$ When $t = 0$, $v = 18 \Rightarrow c = -\ln(3g - 18)$ $\frac{1}{30}t = \ln\frac{3g-18}{3g-v}$ $t = 30\ln\frac{3g-18}{3g-v}$	**M1** **A1A1** **B1** **A1**	 **5**	M1: Separation of variables. A1: Correct LHS. A1:Correct RHS. Condone missing constant. B1: Correct constant. (-2.4336… or AWRT -2.4 or -ln(11.4) or $e^c = \frac{5}{57}$) or $e^{-c} = \frac{57}{5}$ A1: Correct final answer. Accept $t = 30\ln\left(\frac{11.4}{3g-v}\right)$ or $t = -30\ln\left(\frac{3g-v}{11.4}\right)$ or $t = 30\ln\left(\frac{57}{5(3g-v)}\right)$ oe Do not accept $t = 73.008 - 30\ln(3g - v)$
(iii)	When $v = 22 \Rightarrow t = 30\ln\frac{3g-18}{3g-22}$ When $v = 18$ t=0 Time taken is = $30\ln\frac{3g-18}{3g-22}$ = 12.964.. sec = 13.0 sec	**M1** **B1** **A1**	 **3**	M1: Substituting $v = 22$ (must be a log or exp term) B1: Using v = 18 when t=0. PI A1: Correct time. Condone 13
	Total		**13**	

Q	Solution	Mark	Total	Comment
6 (a)	$T_A = \frac{mu^2}{a} - mg$ $T_B = \frac{mv^2}{a} + mg$ $\frac{mu^2-mag}{mv^2+mag} = \frac{T_A}{T_B} = \frac{5}{7}$ $5v^2 + 5ag = 7u^2 - 7ag$ [1] C of E ; $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + 2mag$ $v^2 = u^2 + 4ag$ Hence from [1] $5u^2 + 25ag = 7u^2 - 7ag$ $32ag = 2u^2$ $u = 4\sqrt{ag}$	**M1** **A1** **M1A1** **M1** **A1**	 **6**	M1: Resolving correctly at *A* or *B*. A1: Correct T_A and T_B M1: Use of conservation of energy. Accept any height. A1: Correct equation. M1: Any correct equation linking *ag* and u^2 A1: Correct *u*. Accept $\sqrt{16ag}$
(b)	$v^2 = 20\,ag$ $v = 2\sqrt{5ag}$ Ratio u : v = 2: $\sqrt{5}$ or $2\sqrt{5}$: 5	 **M1** **A1**	 **2**	 M1: Correct *v*. A1: Any correct ratio in any correct form. Do not accept decimals.
	Total		**8**	

Q	Solution	Mark	Total	Comment
7(a)	EPE in *AC* is zero. EPE in *BC* is $\frac{240\times2^2}{6}$ = 160J Total EPE is 160 J	**B1** **B1**	**2**	B1: Zero EPE implied for one string. Must be stated. B1: Shows how to obtain 160. AG
(b)	$\frac{1}{2}\times8v^2$+ EPE [of AC] + EPE [of BC] + work done by friction = 160 Work done by friction = $8g\mu x$ EPE of AC = $\frac{160\times x^2}{4}$ EPE of BC = $\frac{240\times(2-x)^2}{6}$ $4v^2+\frac{160\times x^2}{4}+\frac{240\times(2-x)^2}{6}+8g\mu x = 160$ $4v^2 + 40x^2 + 40(4 - 4x + x^2) + 8g\mu x = 160$ $v^2 = 40\,x - 20x^2 - 2g\mu x$ v = $(40\,x - 20x^2 - 2g\mu x)^{0.5}$	**M1** **B1** **B1** **A1** **A1**	**5**	M1: Energy equation with correct KE and including 160 and at least one EPE. B1: Correct friction term. B1: Both EPEs correct. A1: All terms correct with correct signs. A1: Correct expression for *v*.
(c)	Differentiation of any quadratic [wrt *t* or wrt *x*} $2v\frac{dv}{dt}=\frac{dx}{dt}$(40-40*x*-2gμ) *At max speed* 40-40*x*-2*gμ* = 0 $x = 1-\frac{g\mu}{20}$ OR Max speed of v is when $x = -\frac{b}{2a}$ $x = \frac{40-2g\mu}{2\times20} = 1-\frac{g\mu}{20}$ OR (using forces) $\frac{160x}{2}+8g\mu=\frac{240(2-x)}{3}$ $x = 1-\frac{g\mu}{20}$	**M1** **A1** **(M1)** **(A1)** **(M1)** **(A1)**		M1: Derivative equated to zero. A1: Correct expression. M1: Uses equal roots of a quadratic. A1: Correct expression. M1: Correct equation for zero resultant force. A1: Correct expression.
	Total	**9**	**9**	

Q	Solution	Mark	Total	Comment
8				
		B1		B1 for S clearly through the centre of hemisphere
		B1		B1 for force at R clearly perpendicular to rod
	$PR = 2r\cos 30° = \sqrt{3}r$	**B1**		B1: Correct length of PR.
	Resolving along the rod			
	S cos 30 = *mg* sin 30	**M1**		M1: Resolving to find *S* or *T*.
	S = $\frac{mg}{\sqrt{3}}$	**A1**		A1: Correct expression for *S* or *T*.
	Moments about *R*			
	S. 2*r*cos30.sin30	**M1**		M1: Taking moments about *P* or *R*. in terms of l, r or PR
	= *mg*(2*r*cos 30 - $\frac{1}{2}$ *l*) cos 30	**A1**		Note moments about *P* gives: $T \times 2r\cos 30° = mg \times \frac{l}{2}\cos 30°$
	4*rS* sin30 = *mg*(4*r*cos30 –*l)*			A1: Correct moment equation.
	4*r*. $\frac{mg}{\sqrt{3}}$ .sin30 = *mg*(4*r*cos30 –*l*)			
	$\frac{2r}{\sqrt{3}} = 2\sqrt{3}\ r - l$	**A1**		A1: Correct equation containing only *r* and *l*.
	$\frac{4r}{\sqrt{3}} = l$			
	$r = \frac{\sqrt{3}}{4}l$	**A1**	**9**	A1: Correct expression for *r*.
	Total		**9**	